1. A box contains 10 coins, of which 5 are fair and 5 are biased to land heads with probability 0.7. A coin is drawn from the box and tossed once.
   (a) What is the chance that it will land a head?
   (b) Suppose that the coin drawn landed a head. Given this information, what is the conditional probability that if I draw another coin from the box(without replacing the first coin), then that coin will be a fair coin ?

2. Let $X_1, \ldots X_n, \ldots$ be a sequence of independent and identically distributed random variables, with $E(X_i) = 10$ and $V(X_i) = 1$.
   (a) State the weak law of large numbers for the above mentioned sequence $X_1, \ldots X_n, \ldots$.
   (b) Let $T_n = \frac{10}{n}\sum_{i=1}^{n}(30+X_i)$. Does T_n converge to something and if so what is type of convergence ?

3. Let $Z \sim$ Normal $(0,1)$. Let $X = Z^2$ be a random variable with moment generating function $M_X(t)$. Let Y be another random variable with moment generating function $M_Y(t)$. Suppose $M_X(t) = M_Y(4t)$, then
   (a) Find the relationship between X and Y.
   (b) Find the probability density function of Y.

4. Solve the following questions and giving **reasons** for your answer.
   (a) Let X be a discrete random variable. Which of the following functions can represent the distribution function F of X:-

(i)	**(ii)**	**(iii)**
$F(x) = \begin{cases} 0 & x \leq -1 \\ 0.6 & -1 < x < 1 \\ 1 & x \geq 1 \end{cases}$	$F(x) = \begin{cases} 0 & x < 0 \\ 0.5 & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$	$F(x) = \begin{cases} 0 & x < 0 \\ 0.4 & 0 \leq x < 1 \\ 0.3 & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$

   (b) Decide whether the following statement is true:- "If A, B, and C are pairwise independent events then they are independent events."
   (d) Let $X \stackrel{d}{=}$ Binomial $(36, \frac{1}{2})$. Let $\Phi(t) = \int_{-\infty}^{t} dt \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$. Find the r, s, t, u, v so that the following approximations are valid:
   (i) $P(3 \leq X \leq 20) \approx \Phi(r) - \Phi(s)$
   (ii) $P(X = 20) \approx \dfrac{e^{-t}u^{20}}{v}$

5. Suppose the number of earthquakes (X) that occur in a year, anywhere in the world, is a Poisson random variable with mean λ. Suppose that the probability that any given earthquake has magnitude at least 5 on the Richter scale is p. Let B be the number of earthquakes in a year of magnitude at least 5. Find the distribution of B.

6. Let X and Y be independent random variables each geometrically distributed with parameter $\frac{3}{4}$.
   (a) Find $P(\min(X,Y) = X)$.
   (b) Find the distribution of $X + Y$.
   (c) Find $P(Y = y | X + Y = z)$.